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An Analytical Approach to Coupled Vibration of Curved Rationalized Girder Bridges and Running Vehicles

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Abstract

This study is intended to develop an analytical approach to simulate the coupled vibration of bridges and running vehicles, and to investigate the dynamic characteristics of rationalized curved bridges. In this paper, a curved twin I-girder bridge with general cross-section is adopted for the analysis. As a preliminary step, a general large dump truck with one axis at front and two axes at rear is modeled as a sprung-mass system with 2-degree-of-freedom for the vehicle-bridge interaction analysis. The coupled vibration of the interaction system is formulized and a computer program is developed. Eigenvalue analyses are carried out for different types of bridges to find acceptable torsional stiffness. The coupled vibration of the running vehicle and the bridge is simulated using the developed analytical approach, based on which the dynamic responses of the curved bridge are evaluated.

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Keywords: Curved girder bridge, Vehicle-bridge interaction analysis, Eigenvalue analysis.

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1. Introduction

In recent years, rationalized girder bridges such as twin-girder bridges have been gradually put into practical use due to the cost reduction demands (Yoon et al. 2005). This type of bridge was designed and constructed in France in early 1960s and in Japan in 1990s (kim et al. 2004). However, the low torsional stiffness of the rationalized girder bridge is concerned, especially for curved ones. Therefore, the dynamic characteristics of the rationalized girder bridges should be fully investigated to ensure their structural safety. On the other hand, the traffic induced-vibration problems due to the recently growing traffic loads are also important issues. The low torsional stiffness may lead to excessive structural vibration that may cause fatigue or environmental problems, etc. In order to discuss such problems, it is necessary to theoretically clarify the phenomena at first.

The purpose of this study is to develop an analytical approach to simulate the coupled vibration of curved bridges and running vehicles, which can be used to investigate the dynamic issues mentioned above. In this paper, a curved twin I-girder bridge with general cross-section is adopted for the analysis. As a preliminary step, a general large dump truck with one axis at front and two axes at rear is modeled as a sprung-mass dynamic system with 2-degree-of-freedom (2-DOF). The coupled vibration of the interaction system is formulized and a computer program is developed. To investigate the basic characteristics of the rationalized bridge and to find acceptable torsional stiffness, eigenvalue analyses of the bridges with different types of reinforcing crossbeams are carried out and compared with each other at first. Then, the coupled vibration of the running vehicle and the bridge is simulated using a developed analytical approach to discuss the dynamic responses of the curved bridge.

2. Bridge Models And Eigenvalue Analyses

2.1 Rationalized girder bridge model

In this study, a curved twin I-girder bridge with general cross-section as shown in Figure 1 is adopted, which has a span length of 50 m. This bridge has a PC slab with 0.3 m thickness. The middle crossbeams with I-shaped cross-section are perpendicularly connected to the centers of the main girders in vertical direction, while the end ones are connected at the upper sides of the girders. The crossbeams are aligned along the main girders in longitudinal direction at an interval of 5 meter. The dimensions of I-shaped beam cross-sections used in this analysis are given in Table 1. The material properties of the PC slab and steel members used in this model are shown in Table 2. The boundary condition of this bridge is simply-support.

The three dimensional beam element model of the twin I-girder bridge superstructure described above is shown in Figure 2 and designated as the basic model. To find acceptable torsional stiffness, two reinforcement countermeasures to increase the torsional stiffness of the superstructure are designed and modeled with FE elements as shown in Figure 3 and Figure 4. Figure 3 indicates the measure to install diagonal reinforcing I-shaped beams (Diagonal reinforcement) between the perpendicular ones, which have the same cross-section as the middle crossbeams. Figure 4 shows another conceived measure to add cross bracing beams to the bottoms of main girders (Bottom bracing reinforcement), whose cross-section properties are also indicated in Table 1. The eigenvalue analyses of these three models are carried out to evaluate their difference of the torsional stiffness.

2.2 Eigenvalue estimation

For the three bridge models described above, eigenvaule analyses are performed using QR method. The primary vertical mode (V_1) and the primary torsional mode (T_1) as well as their ratio of the models with curvature radius R=∞ (straight) and R=200 m (curved) are respectively shown in Table 3 and Table 4. The parameter of frequency ratio, f_{T1}/f_{V1}, is used to evaluate the torsional stiffness. In general, this frequency ratio of a steel bridge with general open cross-section is around 2.0. As shown in Table 3 and 4, the torsional frequency of the basic model is rather low. On the other hand, the frequency ratios of the diagonal reinforcement are 2.3644 and 2.4670 corresponding to straight and curved models, indicating that the torsional stiffness of this model is high enough. The reason is considered as that the girders, crossbeams and the slabs form a pseudo box girder cross-section. Furthermore, the bottom bracing reinforcement model also has high frequency ratios of 2.9003 and 3.0009, which is more effective to increase the torsional stiffness. However, this measure is conceived without estimation on the economical demands and convenience for future maintenance, which are important factors in actual applications.

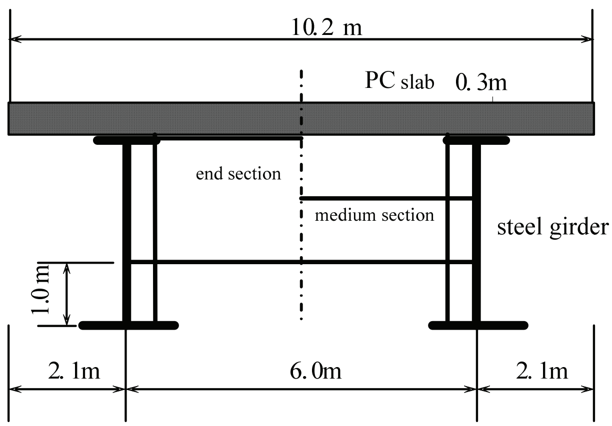


Figure 1: Cross-section of twin I-girder bridge

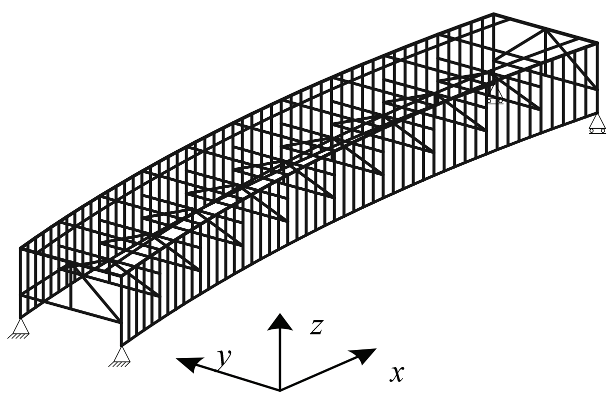


Figure 2: Basic FE model

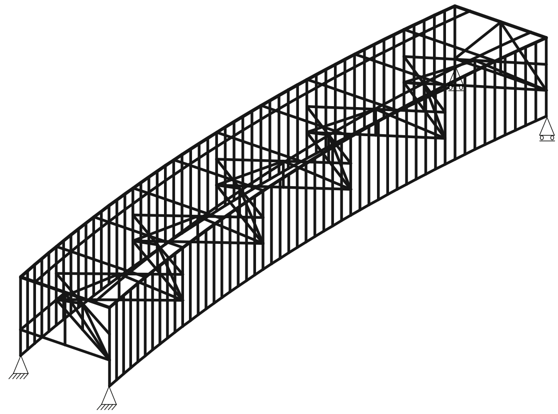


Figure 3: Diagonal reinforcement model

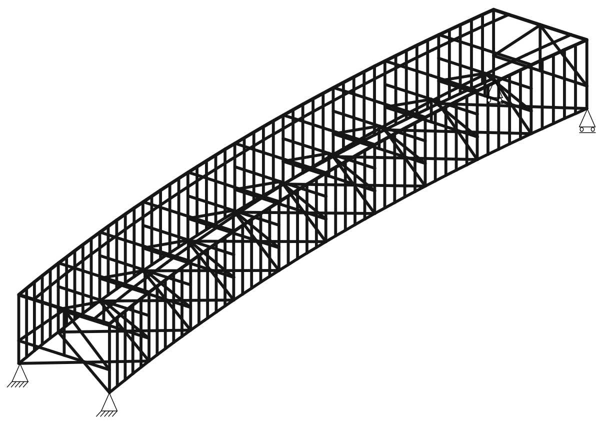


Figure 4: Bottom bracing beam model

3. Coupled Vibration Analysis of Train-Bridge System

3.1 Vehicle-bridge interaction analysis procedure

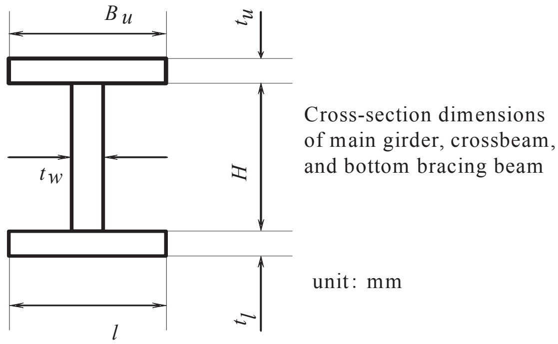
In this paper, dynamic responses of the vehicle-bridge interaction system are simulated using a developed computer program based on the formulization process described below. Modal analysis technique is applied to the simultaneous dynamic differential equations of the structure. The Newmark's β step-by-step numerical integration method is applied to solve the dynamic differential equations.

3.3.1 Formulization of vehicle vibration

For the analysis of running vehicle-bridge interaction problem, a general large dump truck with one-axis at front and two-axis at rear are adopted. As a preliminary stage, a simple vehicle model is used to perform the coupled vibration analysis. As shown in Figure 5, the truck is modeled as sprung-mass dynamic system with 2-degree-of-freedom, where variables z_j and θ_j respectively indicate the DOFs of the bouncing and pitching of the vehicle body. The vehicle properties are shown in Table 5. Then, based on

D'Alember's principle and force equilibrium on each degree-of-freedom, the vehicle-bridge interaction can be formulized as follows.

Table 1: Cross-section properties of bridge member



	Main girder	End crossbeam	Middle crossbeam	Bottom bracing beam
B_u	500	300	300	250
T_u	30	25	25	24
H	3000	2000	1000	250
t_w	24	16	16	16
B_l	800	300	300	None
t_l	50	25	25	None

Table 2: Properties of bridge members

	Young's modulus E [N/mm^2]	Poisson's ratio μ	Unit weight w [kN/m^3]
PC slab	2.857×10^5	0.2	24.5
Steel member	2.000×10^4	0.3	77.0

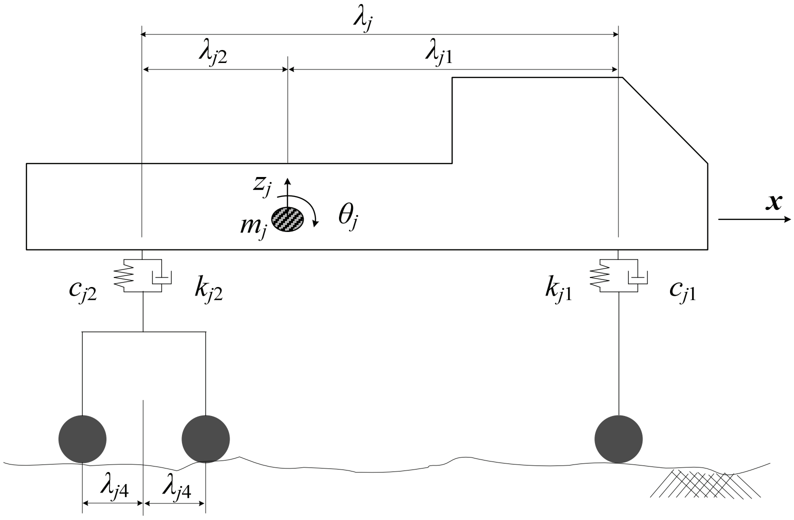


Figure 5: 2-DOF vehicle model

Table 3: Natural frequencies (R=∞)

	f_{V1} (Hz)	f_{T1} (Hz)	f_{T1}/f_{V1}
Basic model	2.4954	3.3374	1.3374
Diagonal reinforcement	2.5851	6.1122	2.3644
Bottom bracing reinforcement	2.4947	7.2354	2.9003

Table 4: Natural frequencies (R=200m)

bridge model	f_{V1} (Hz)	f_{T1} (Hz)	f_{T1}/f_{V1}
Basic model	2.2872	3.6532	1.5972
Diagonal reinforcement	2.5161	6.2073	2.4670
Bottom bracing reinforcement	2.4669	7.4030	3.0009

Bouncing of the vehicle body can be expressed as follows.

$$m_j \ddot{z}_j + \sum_{l=1}^{lx(j)} v_{jl}(t) = 0 \tag{1}$$

Pitching of the vehicle body can be expressed as follows.

$$I_j \ddot{\theta}_j + \sum_{l=1}^{lx(j)} (-1)^l \lambda_{jl} v_{jl}(t) = 0 \tag{2}$$

where,

$$v_{jl}(t)=k_{jl}\left\{z_j+(-1)^l\lambda_{jl}\theta_j-\frac{1}{kx(l)}\sum_{k=1}^{kx(l)}w_{jlk}\right\}+c_{jl}\left\{\dot{z}_j+(-1)^l\lambda_{jl}\dot{\theta}_j-\frac{1}{kx(l)}\sum_{k=1}^{kx(l)}\dot{w}_{jlk}\right\} \tag{3}$$

Table 5: Properties of the vehicle model

	Definition	Notation	Value
Parameter	Mass of vehicle body	m_j	17.87 t
	Front unsprung mass	m_{j1}	0.40 t
	Rear unsprung mass	m_{j2}	0.60 t
	Moment of inertia of sprung mass	I_j	544.21 kN·s^2·m
	Spring constant of front suspension	k_{j1}	668.36 kN/m
	Spring constant of rear suspension	k_{j2}	5328.26 kN/m
	Damping coefficient of front suspension	c_{j1}	11.76 kN·s/m
	Damping coefficient of rear suspension	c_{j2}	27.83 kN·s/m
Geometry	Distance between front and suspension centers	λ_j	4.00 m
	Distance between front axle and body centroid	λ_{j1}	2.95 m
	Distance between rear suspension and body centroid	λ_{j2}	1.05 m
	1/2 distance of tandem axle	λ_{j4}	0.68 m

Here, suffixes j, l and k are used to define the variants and parameters used in the vehicle model, and are specified as follows. j indicates the vehicle number. l = 1 or 2 denotes the front or rear suspension; k = 1 or 2, the front or rear wheel axle at the front or rear suspension, while in this model only one axle (k = 1) exists at the front suspension. $lx(j)$ is the function of vehicle number, which defines the number of suspensions in that vehicle. $kx(l)$ is the function of vehicle suspension number, which defines the number of axles in that suspension. $v_{jl}(t)$ expresses the forces due to the spring deformation between the vehicle body and suspensions.

w_{jlk} expresses the displacement at the contact point of the tire and the road surface, which is a combination of the slab deflection and the road surface roughness.

The wheel load is shown in the below equation, where g is the acceleration of gravity.

$$P_{jlk}(t)=-\frac{1}{kx(l)}g\left\{\left(1-\frac{\lambda_{jl}}{\lambda_j}\right)m_j+m_{jl}\right\}+\frac{1}{kx(l)}v_{jl}(t) \tag{4}$$

Expanding and substituting the above equations, matrix form of the formulization can be derived as below, where $\boldsymbol{M}_v$, $\boldsymbol{C}_v$, $\boldsymbol{K}_v$ and $\boldsymbol{F}_v$ are mass, damping, stiffness matrixes and external force vector respectively.

$$\boldsymbol{M}_v\ddot{\boldsymbol{w}}_v+\boldsymbol{C}_v\dot{\boldsymbol{w}}_v+\boldsymbol{K}_v\boldsymbol{w}_v=\boldsymbol{F}_v \tag{5}$$

3.1.2 Formulization of bridge vibration

The dynamic differential equations of the bridge can be derived as follows, based on FEM theories and D'Alembert's Principle.

$$\boldsymbol{M}_b \ddot{\boldsymbol{w}}_b + \boldsymbol{C}_b \dot{\boldsymbol{w}}_b + \boldsymbol{K}_b \boldsymbol{w}_b = \boldsymbol{F}_b \tag{6}$$

where, $\boldsymbol{M}_b$, $\boldsymbol{C}_b$, $\boldsymbol{K}_b$ and $\boldsymbol{w}_b$ respectively denote the bridge mass, damping, stiffness matrices and the nodal displacement vector. Herein, the damping matrix $\boldsymbol{C}_b$ is calculated by Rayleigh damping The external force vector, $\boldsymbol{F}_b$, can be expressed as follows, where, $P_{jlk}(t)$ and $\boldsymbol{\Psi}_{jlk}(t)$ respectively denote the wheel load and the distribution vector, while h is the total vehicle number.

$$\boldsymbol{F_b} = \sum_{j=1}^{h} \sum_{l=1}^{lx(j)} \sum_{k=1}^{kx(l)} \boldsymbol{\Psi}_{jlk}(t) P_{jlk}(t) \tag{7}$$

Applying the modal analysis technique to the bridge system, the structural displacement vector, $\boldsymbol{w}_b$, can be expressed as follows using eigenvectors $\boldsymbol{\varphi}_i$ and generalized coordinates q_i, where, subscript i is the mode number and n indicates the highest one to be considered.

$$\boldsymbol{w}_b = \sum_{i=1}^{n} \boldsymbol{\varphi}_i q_i = \boldsymbol{\Phi} \cdot \boldsymbol{q} \tag{8}$$

Substitute $\boldsymbol{w}_b$ into the bridge vibration equation will obtain the following equation.

$$\boldsymbol{M}_b \boldsymbol{\Phi} \ddot{\boldsymbol{q}} + \boldsymbol{C}_b \boldsymbol{\Phi} \dot{\boldsymbol{q}} + \boldsymbol{K}_b \boldsymbol{\Phi} \boldsymbol{q} = \boldsymbol{F}_b \tag{9}$$

Multiply $\boldsymbol{\Phi}^T$ to both sides,

$$\boldsymbol{\Phi}^T \boldsymbol{M}_b \boldsymbol{\Phi} \ddot{\boldsymbol{q}} + \boldsymbol{\Phi}^T \boldsymbol{C}_b \boldsymbol{\Phi} \dot{\boldsymbol{q}} + \boldsymbol{\Phi}^T \boldsymbol{K}_b \boldsymbol{\Phi} \boldsymbol{q} = \boldsymbol{\Phi}^T \boldsymbol{F}_b \tag{10}$$

According to the orthogonality of eigenvectors, and assuming $\boldsymbol{\varphi}_i^T \boldsymbol{F}_b = F_i$, the bridge equation corresponding to each mode can be expressed as follows by generalized coordinates.

$$M_i \ddot{q}_i + C_i \dot{q}_i + K_i q_i = F_i \tag{11}$$

3.2 Analytical evaluation on coupled vibration

Using the developed analytical approach introduced above, the running vehicle-bridge interaction analyses are carried out. Here, only the dynamic response of the basic model and the diagonal reinforcement model are used for evaluation. The radii of curvature of both models are set as 200 m. The number of running vehicle is one, and its velocity is 40 km running along a straight lane. The output point of each model is the center node of the main girder in longitudinal direction on the running side, whose static defection is considered to be the largest. The acceleration responses and the dynamic displacements of the output points are respectively shown in Figure 6 and Figure 7, and the comparison of the maximum accelerations and deflections are given in Table 6.

For these two models, both the maximum acceleration and deflection of the basic model are rather larger than those of the diagonal reinforcement model. Apparently, this phenomenon is owing to the difference of the torsional stiffness of the superstructure. From this results, it can be confirmed that by increasing the torsional stiffness of the twin I-girder bridges, the intensity of the traffic-induced vibration of the bridge can be reduced effectively, which is important for the fatigue and environmental vibration problems, etc.

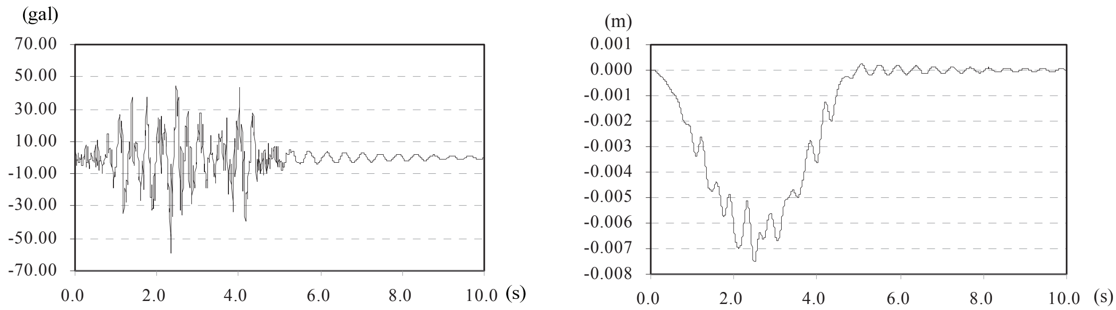


Figure 6: Acceleration and displacement responses of basic model

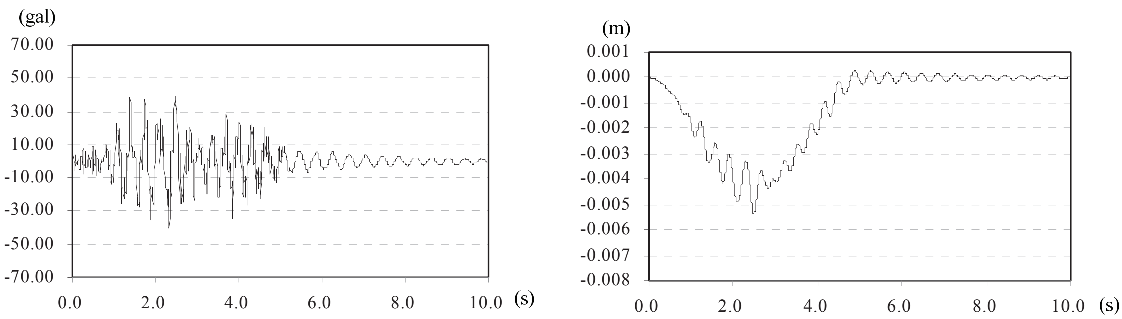


Figure 7: Acceleration and displacement responses of diagonal reinforcement model

Table 6: Maximum acceleration and deflection

	Maximum acceleration	Maximum deflection
Basic model	59.3 gal	7.53 mm
Diagonal reinforcement	40.6 gal	5.36 mm

4. Conclusions

In this paper, different types of twin I-girder bridges are designed and their eigenvalues are compred to find acceptable torsional stiffness. Then, an analytical approach to simulate the running vehicle-bridge coupled vibration problem is formulized and coded, taking advantage of a 2-DOF general large dump truck model and a three-dimensional FE bridge model. The effectiveness of the diagonal reinforcement and bottom bracing reinforcement models to increase the torsional stiffness of the superstructure is confirmed. Such reinforcement is also effective to reduce the traffic-induced dynamic response of the bridge, which is confirmed by the analytical results. For the future works, more elaborated vehicle and bridge models need to be established and the analytical results should be demonstrated via comparison with experiment results or other methods.

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